Hidden yet quantifiable: A lower bound for confounding strength using randomized trials

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Problem Setting

- $\mathbb{P}^{\diamond}$ over $(X, U, Y(0), Y(1), Y, T)$ for $\diamond \in \{\text{rct}, \text{os}\}$
- We observe $D_{\diamond} = \{(X_i, Y_i, T_i)\}_{i=1}^n$ sampled i.i.d from $\mathbb{P}^{\diamond}$

Trade-off between randomized and observational data:

- $\mathbb{P}^{\text{rct}}$ satisfies internal validity: $T \perp\!\!\!\perp (Y(1), Y(0))$
  - $\implies$ we can estimate the ATE $\mu^{\text{rct}} := \mathbb{E}_{\mathbb{P}^{\text{rct}}}[Y(1) - Y(0)]$
  - but the support of $\mathbb{P}_X^{\text{rct}}$ is limited (e.g. no children)
- $\mathbb{P}^{\text{os}}$ covers a broader population: $\text{supp}(\mathbb{P}_X^{\text{rct}}) \subset \text{supp}(\mathbb{P}_X^{\text{os}})$
  - but hidden confounding $\implies$ ATE μ^{os} is not identifiable

How strong is hidden confounding?

- $\mathbb{P}^{\text{os}}$ has confounding strength[1] $\Gamma^\star$ if

$$d_{\text{OR}}\left(\mathbb{P}^{\text{os}}(T \mid X, U), \mathbb{P}^{\text{os}}(T \mid X)\right) = \Gamma^\star$$

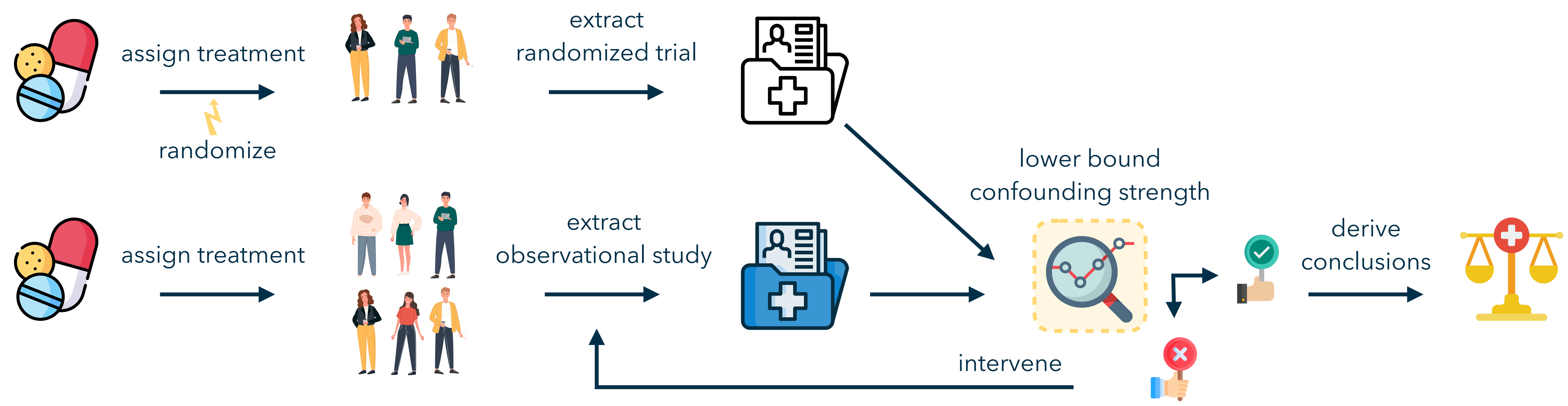
- $\overset{(2)}{\implies} \mathbb{E}_{\mathbb{P}^{\text{os}}}[Y(1) - Y(0) | X] \in [\mu_\Gamma^-(X), \mu_\Gamma^+(X)]$ if $\Gamma \geq \Gamma^\star$

Goal: Can we detect if $\Gamma^\star$ is large enough to affect our conclusions derived from obs. data?

Prior works

- **without rct**[3]: Sensitivity analysis and its critical value $\hat{\Gamma}_{\text{CT}}$
  - ✗ no relation to the true confounding strength $\Gamma^\star$
  - ✓ our work: provides a lower bound on $\Gamma^\star$
- **with rct**[4]: Tests for the null $H_0 : \Gamma^\star > 1$
  - ✗ reject if $\Gamma^\star$ is small $\implies$ too sensitive
  - ✓ our work: test that rejects only if $\Gamma^\star$ is large
- **with rct**[5]: Estimate the bias and correct for it
  - ✗ requires parametric assumptions on the bias structure
  - ✓ our work: no assumptions on the bias structure

Our decision-making pipeline



Method: Detecting Hidden Confounding

- **Goal**: Design a test $\phi_\alpha(\Gamma)$ for

$$H_0(\Gamma) : \mathbb{P}^{\text{os}} \text{ has confounding strength } \leq \Gamma$$

- **How**: $H_0(\Gamma) \overset{\text{transportability}}{\implies} \mu^{\text{rct}} \in \left[\mathbb{E}_{\mathbb{P}^{\text{rct}}}[\mu_\Gamma^-(X)], \mathbb{E}_{\mathbb{P}^{\text{rct}}}[\mu_\Gamma^+(X)]\right]$

1. Estimate: $\hat{\mu}$ using D_{rct}, $\hat{\mu}_\Gamma^-$ and $\hat{\mu}_\Gamma^+$ using D_{os}
2. Bootstrap the estimates to obtain the resp. variances
3. Construct an asymptotically valid two-sided t-test

Corollary: A lower bound for confounding strength

We can then estimate a lower bound for $\Gamma^\star$:

$$\hat{\Gamma}_{\text{LB}} = \inf_{\Gamma}\{\Gamma : \hat{\phi}_\alpha(\Gamma) = 0\}$$

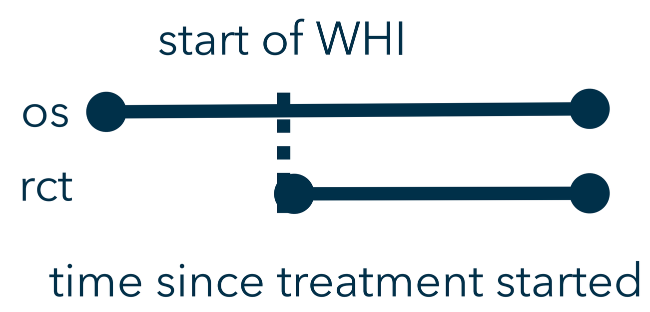
that is asymptotically valid: $\mathbb{P}(\Gamma^\star \geq \hat{\Gamma}_{\text{LB}}) \geq 1 - \alpha + o_{\mathbb{P}}(1)$

References

1. Tan 2006. A distributional approach for causal inference using propensity scores.
2. Kallus et al. 2019. Interval estimation of individual-level causal effects under unobserved confounding.
3. Jin et al. 2023. Sensitivity analysis of individual treatment effects: A robust conformal inference approach.
4. Hussain et al. 2023. Falsification of internal and external validity in observational studies via conditional moment restrictions.
5. Kallus et al. 2018. Removing hidden confounding by experimental grounding.

Real-world Experiments

- **Data:** Women's Health Initiative
  - T := hormone therapy
  - Y := coronary heart disease
  - U := time since treatment



- **Goal**: Detect absence and presence of hidden confounding
- **Our procedure**: `flag` $:= \mathbb{I}\{\hat{\Gamma}_{\text{LB}} > \hat{\Gamma}_{\text{CT}}\}$
  - where e.g. $\hat{\Gamma}_{\text{CT}} := \inf\{\Gamma : 0 \in [\mathbb{E}_{\mathbb{P}^{\text{os}}}[\hat{\mu}_\Gamma^-(X)], \mathbb{E}_{\mathbb{P}^{\text{os}}}[\hat{\mu}_\Gamma^+(X)]]\}$
- **Baseline procedure**: `flag-binary` $= \mathbb{I}\{\hat{\Gamma}_{\text{LB}} > 1\}$

	Coronary heart disease	
treatment started	with the study	before the study
$\hat{\Gamma}_{\text{CT}}$	1.017	1.164
$\hat{\Gamma}_{\text{LB}}$	1.009	1.224
`flag-binary`	1	1
`flag` (ours)	0	1

Bonus: Future work

- Kernelized test to detect confounding even in small subgroups!