Statistical and computational guarantees for the Baum-Welch algorithm

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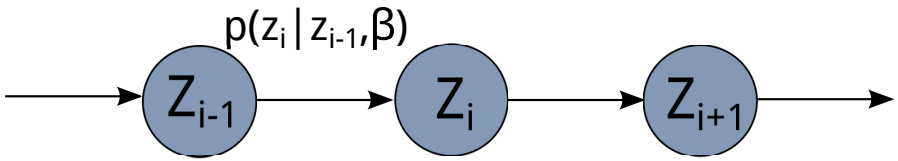
Outline

1. Background
   - Hidden Markov Model and algorithms
   - Classical convergence analysis of the EM algorithm

2. Main results
   - Convergence guarantees for the Baum-Welch algorithm
   - Special case - Gaussian output HMM
   - Discussion

Hidden Markov Models

Parametric Hidden Markov Model (HMM) with discrete latent variables Z_i and observed variables X_i



Model assumptions

- Markov chain $\{Z_i\}_{i\in\mathbb{Z}}$ has a unique stationary distribution, is sufficiently mixing

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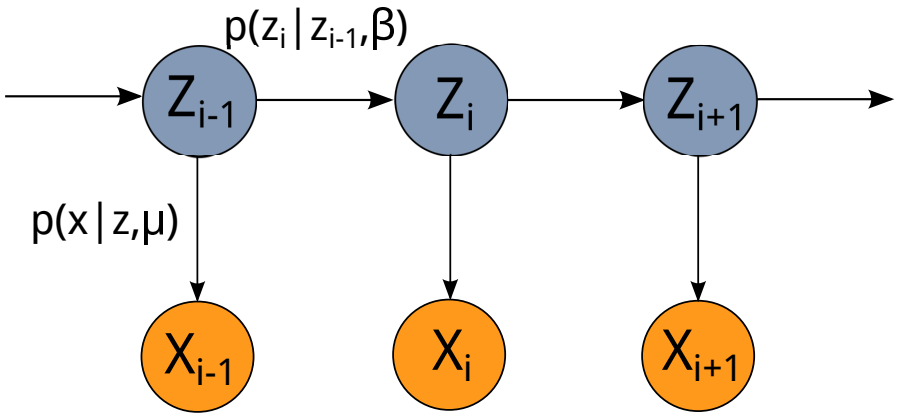

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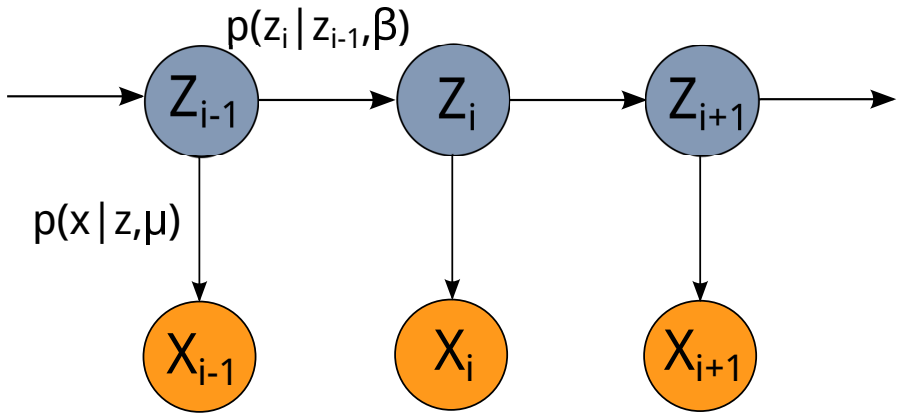

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- HMM parameterized by $\theta = (\beta, \mu)$

Goal: Reconstruct θ from a sequence of observations $X_1^n := X_1, \ldots, X_n$ drawn from the HMM

Hidden Markov Models - Reconstruction algorithms

Algorithms for general HMM reconstruction, e.g.

- Classical approach: Baum-Welch algorithm (Baum et al. 1970) and EM variants (with k-means) (Dasgupta and Schulman 2007)
- Spectral methods (Hsu, Kakade and Zhang 2012)
- Convex programs for parametric-output HMMs (Kontorovich et al. 2013)

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Motivation to analyze EM

- Widely used and easy to implement for many models
- Empirically high precision when initialized correctly
- But lack of theoretical work which explains this behavior (for i.i.d. data Balakrishnan et al. 2014)

Hidden Markov Models - EM algorithm

Recall the complete log likelihood

$$\log p(z_1^n, x_1^n; \theta) = \log \left[\pi_1(z_1) \prod_{i=2}^{n} p(z_i \mid z_{i-1}; \beta) \prod_{i=1}^{n} p(x_i | z_i, \mu) \right]$$
$$= \sum_{i=1}^{n} \log f(z_{i-1}^i; \beta, \mu)$$

EM maximizes the log likelihood

$$\widehat{\theta}_{\text{MLE}} := \arg\max_{\theta} \ell_n(\theta) := \arg\max_{\theta} \left[\log p(x_1^n; \theta) \right]$$
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Key of EM: Optimize over the *expected* complete log likedlihood

EM algorithm for latent variable models

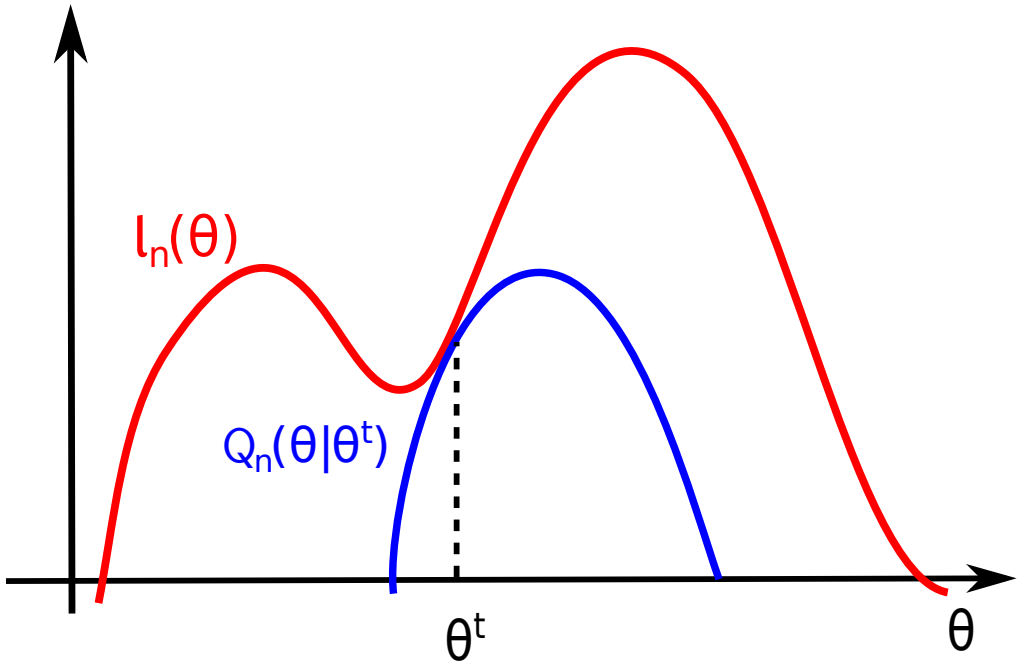
Observe the lower bound for all θ'

$$\begin{aligned}\ell_n(\theta) &\geq \frac{1}{n}\mathbb{E}_{Z_1^n|X_1^n,\theta'}\left[\log p(X_1^n, Z_1^n; \theta)\right] + H(\theta') \\ &=: Q_n(\theta \mid \theta').\end{aligned}$$

with $\ell_n(\theta) = Q_n(\theta \mid \theta)$.

EM algorithm for latent variable models

Using $\ell_n(\theta) \geq Q_n(\theta \mid \theta')$ and $\ell_n(\theta') = Q_n(\theta' \mid \theta')$



Algorithm (EM algorithm)

Initialize with $\widehat{\theta}^0$ *and then iterate until convergence*

$$\widehat{\theta}^{t+1} = \arg\max_{\theta \in \Theta} Q_n(\theta \mid \widehat{\theta}^t)$$

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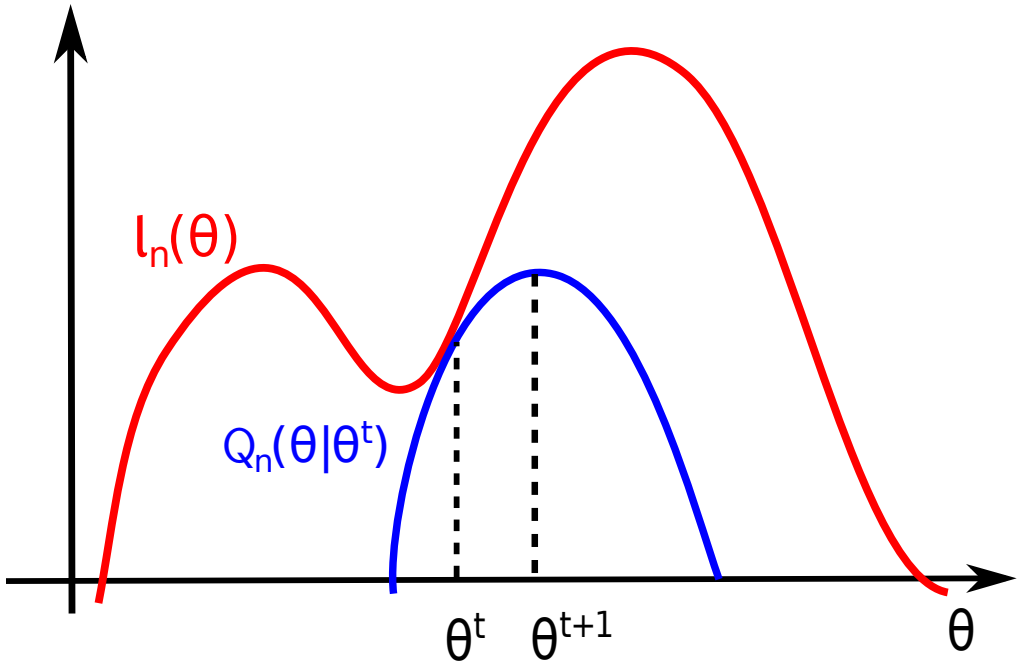

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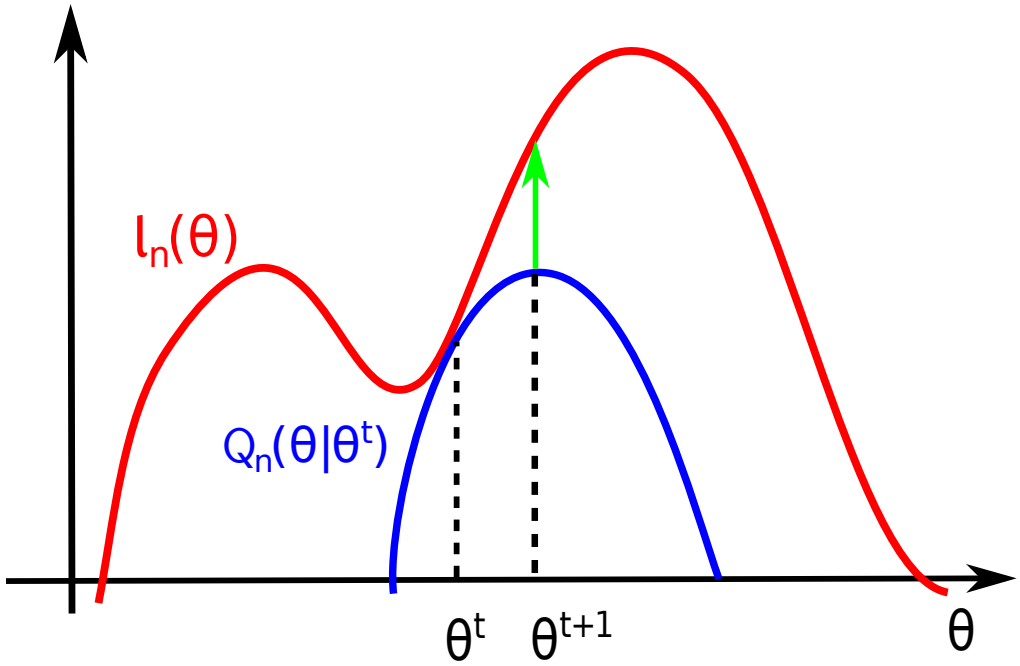

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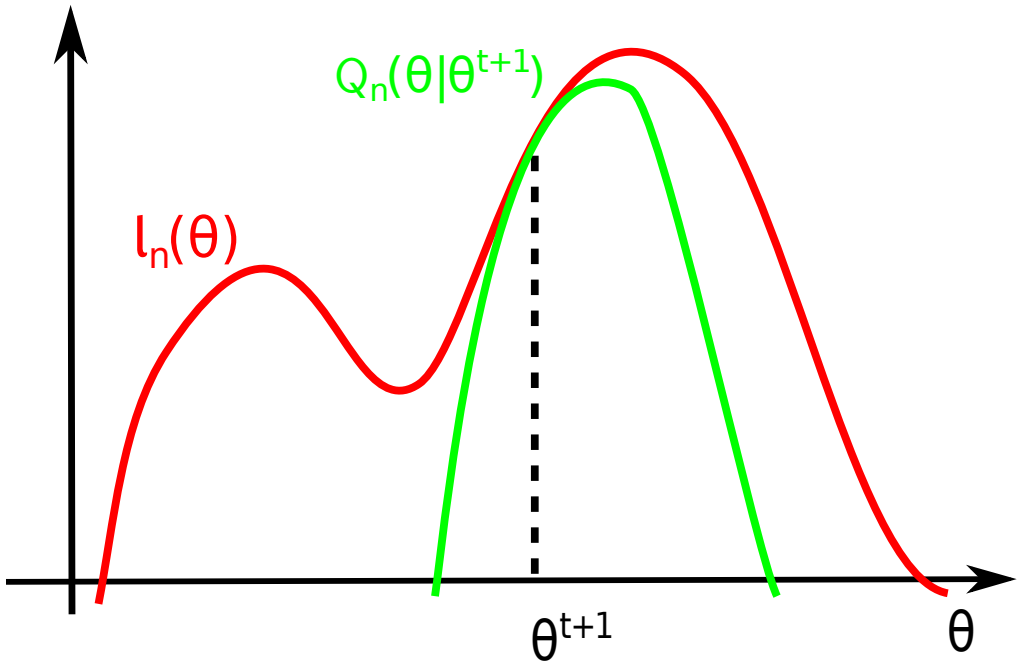

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EM algorithm - classical convergence analysis

Classical EM convergence analysis (Dempster et al. 1977, Wu 1983, Hathaway 1983, Xu and Jordan 1996, Nettleton 1999):

- MLE is a fixed point of Q_n, i.e. $\widehat{\theta}_{\text{MLE}} = \arg\max_\theta Q_n(\theta \mid \widehat{\theta}_{\text{MLE}})$
- Under regularity assumptions $\widehat{\theta}^t$ converges to a stationary point of the sample likelihood
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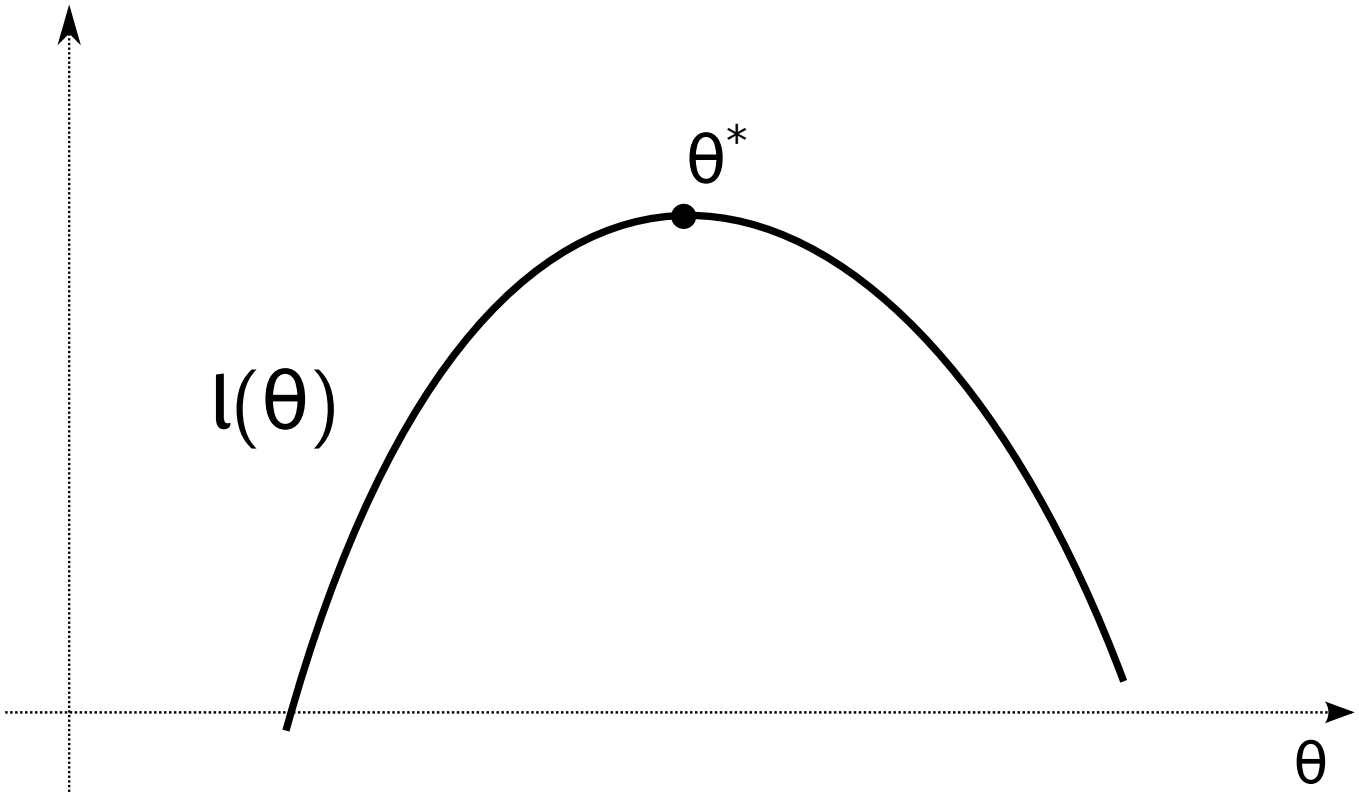
Main caveat: necessary initialization radius or linear convergence rates only given in an **arbitrarily small ball** around the MLE

Classical analysis - caveats



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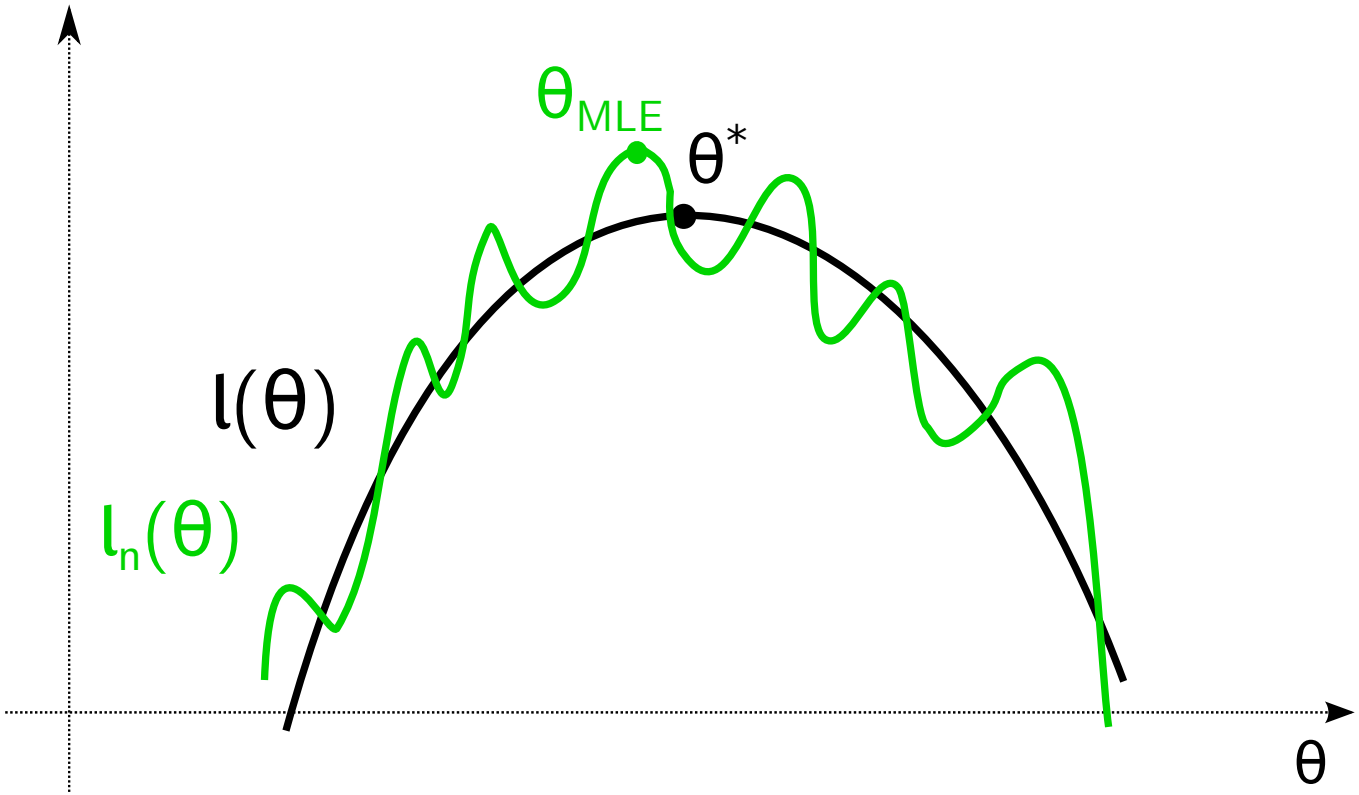

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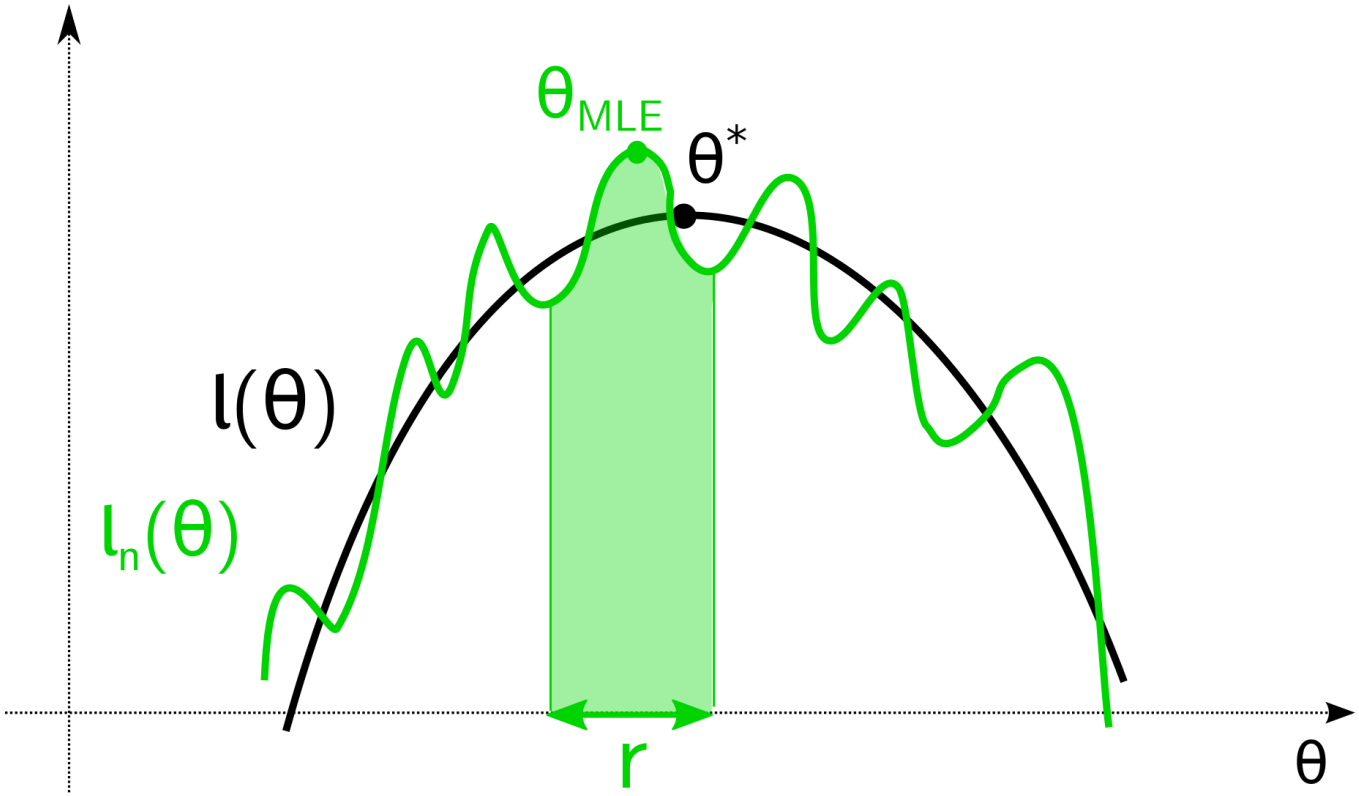

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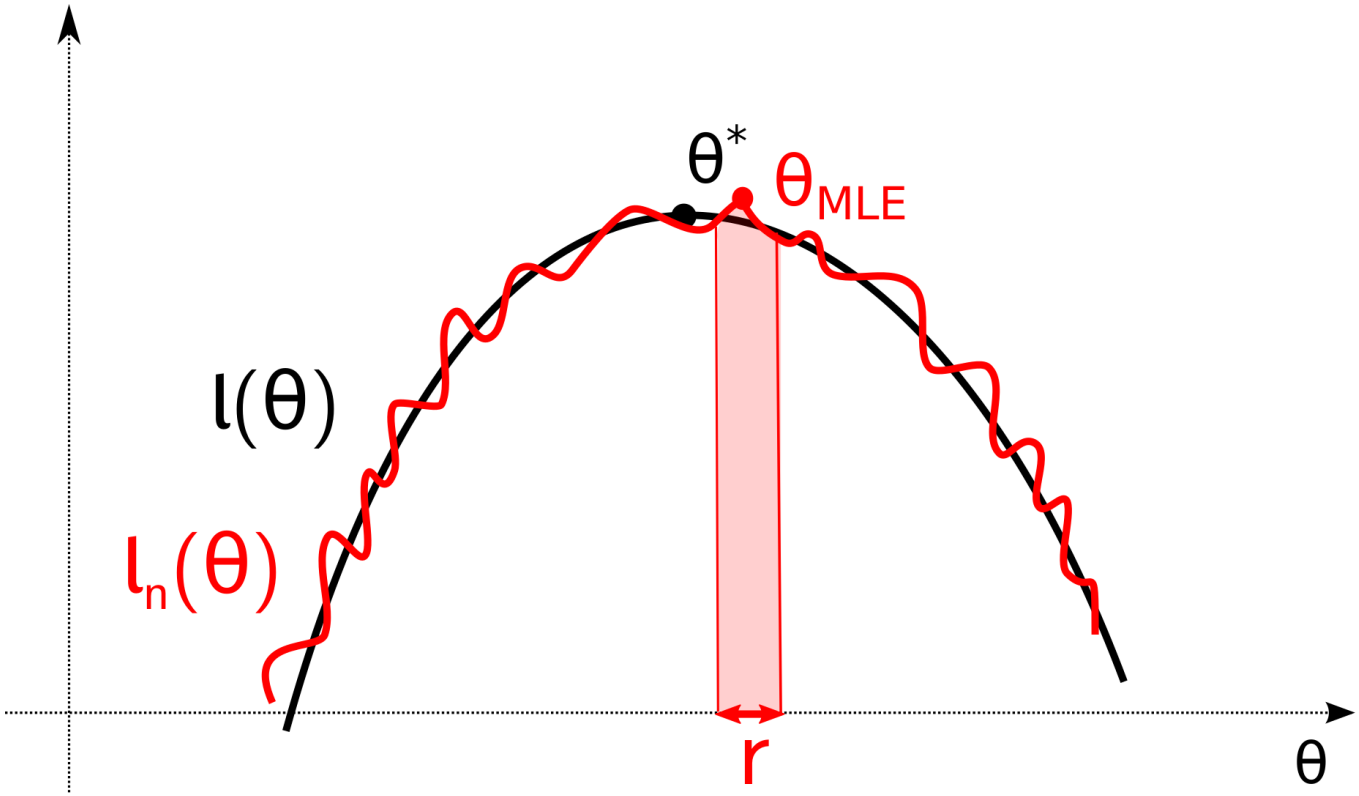

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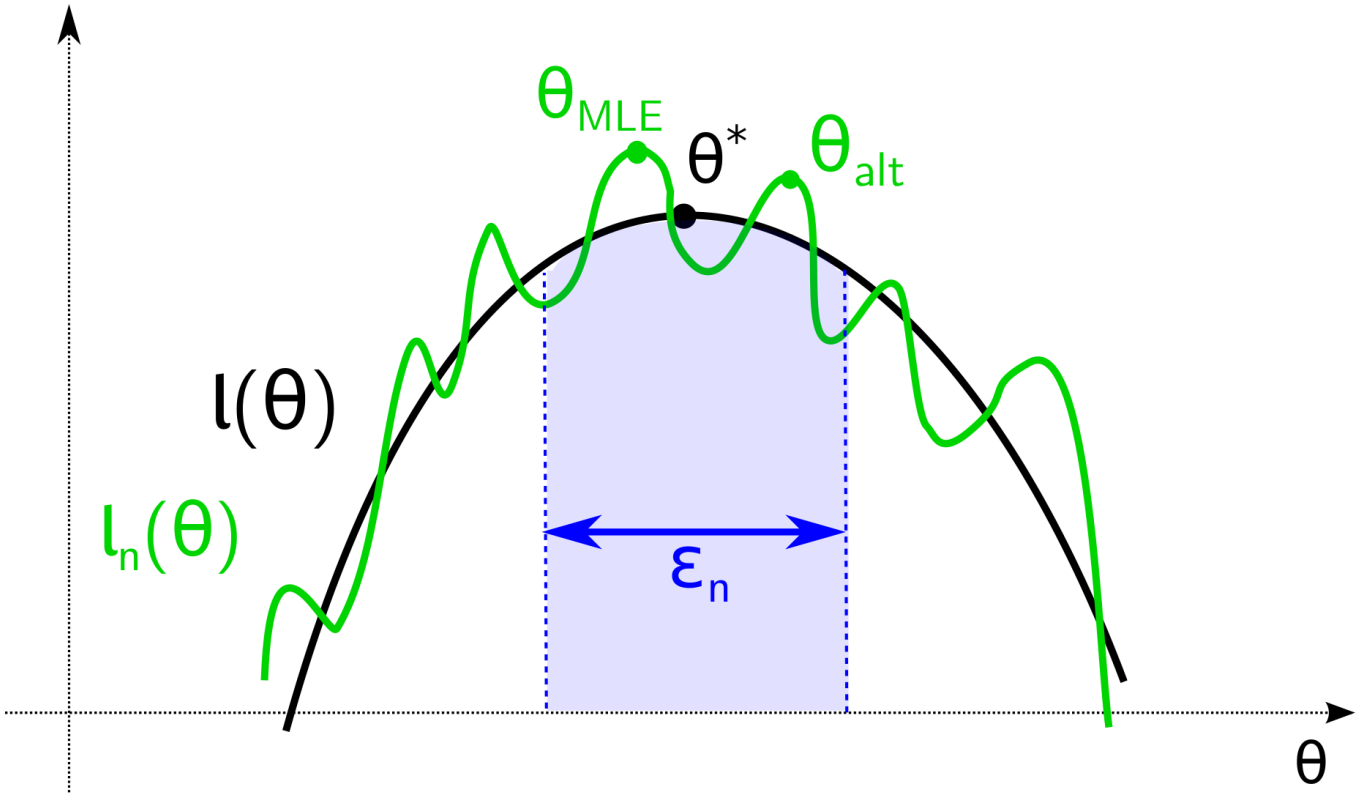

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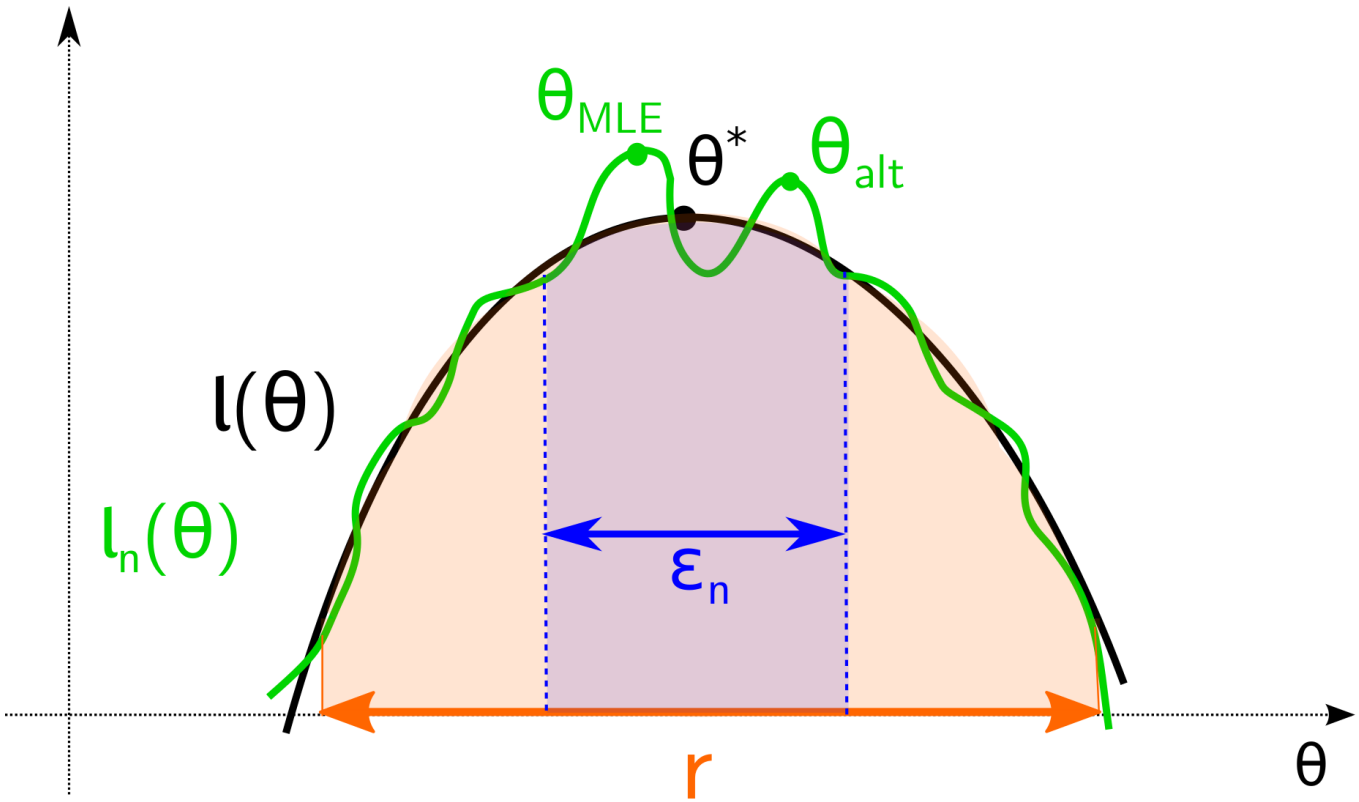
Our contributions



Characterize initialization radius r around θ^* such that

- all stationary points in r are in an ϵ_n ball around θ^*
- Baum-Welch converges linearly to the ϵ_n ball

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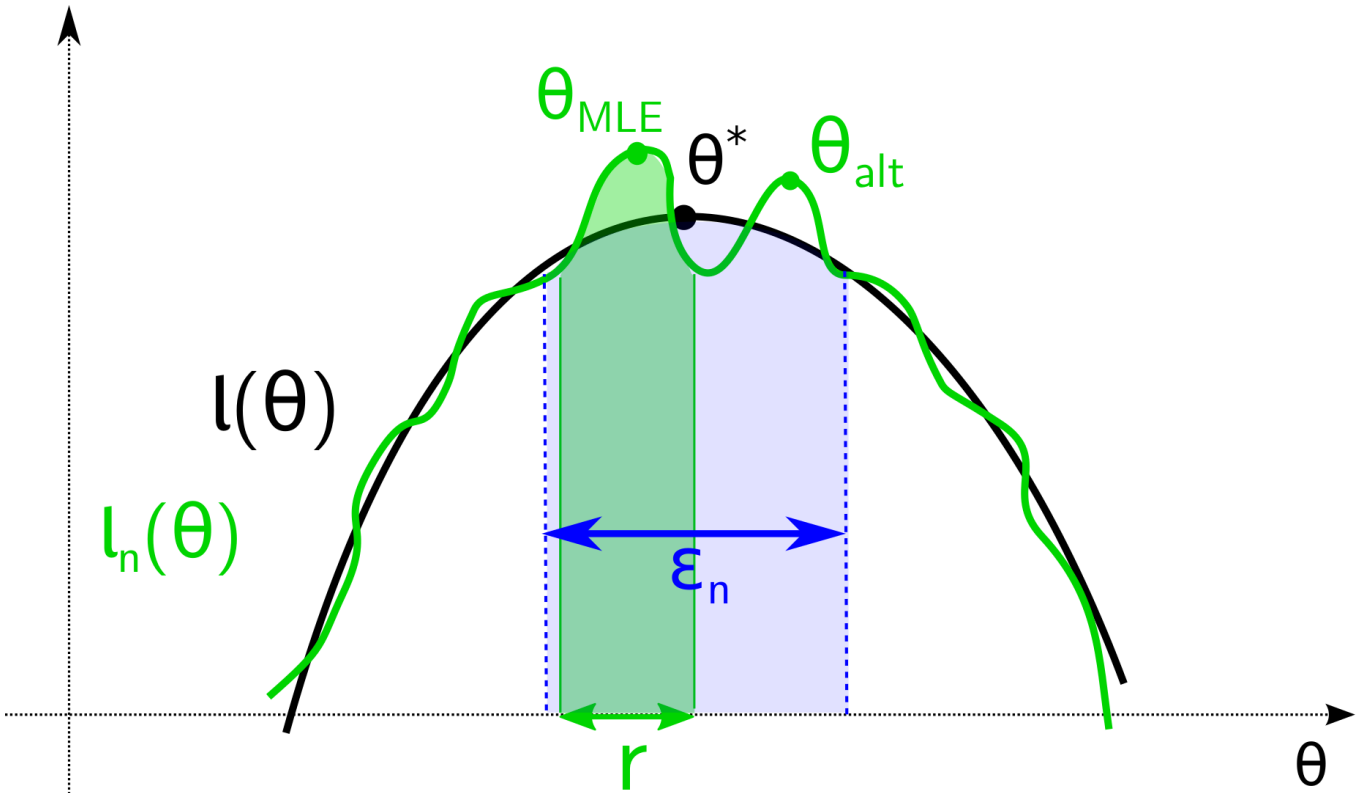

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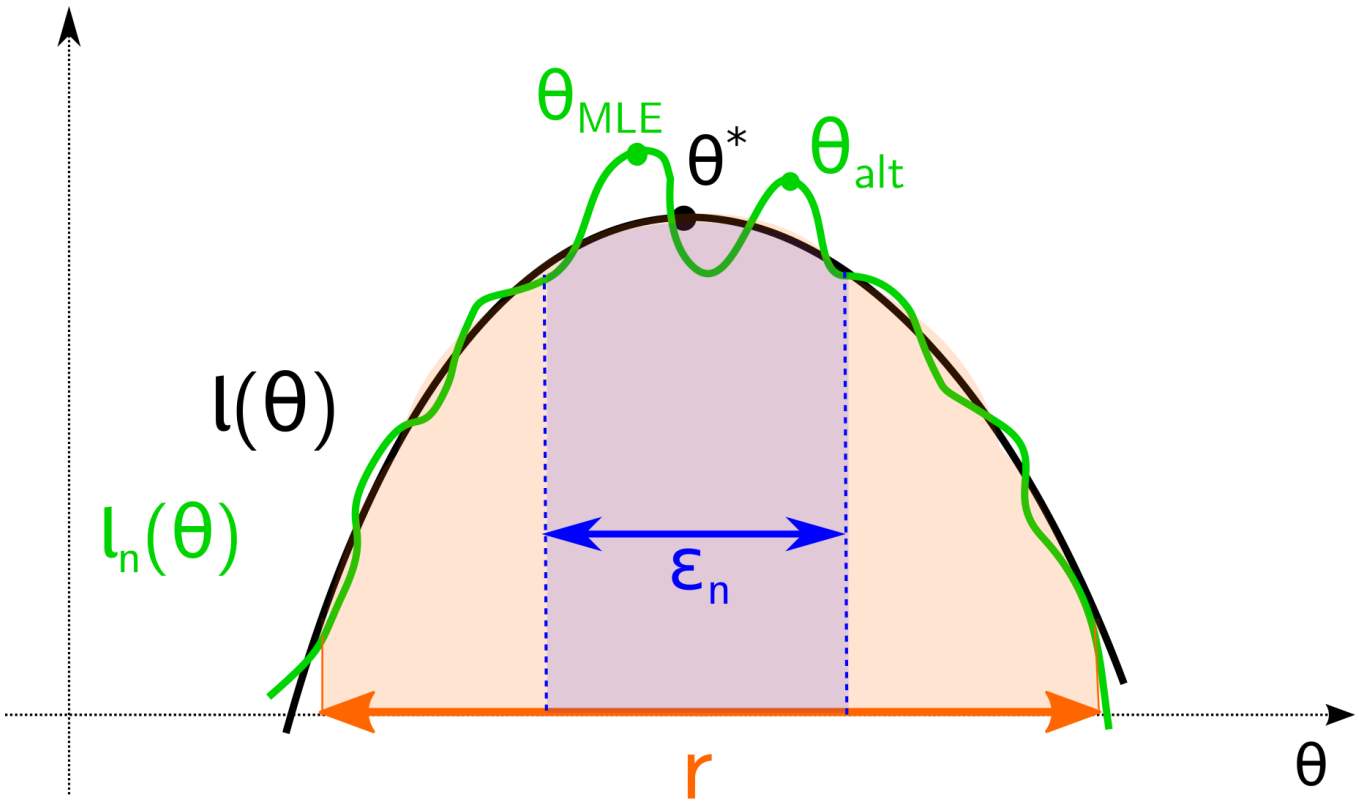

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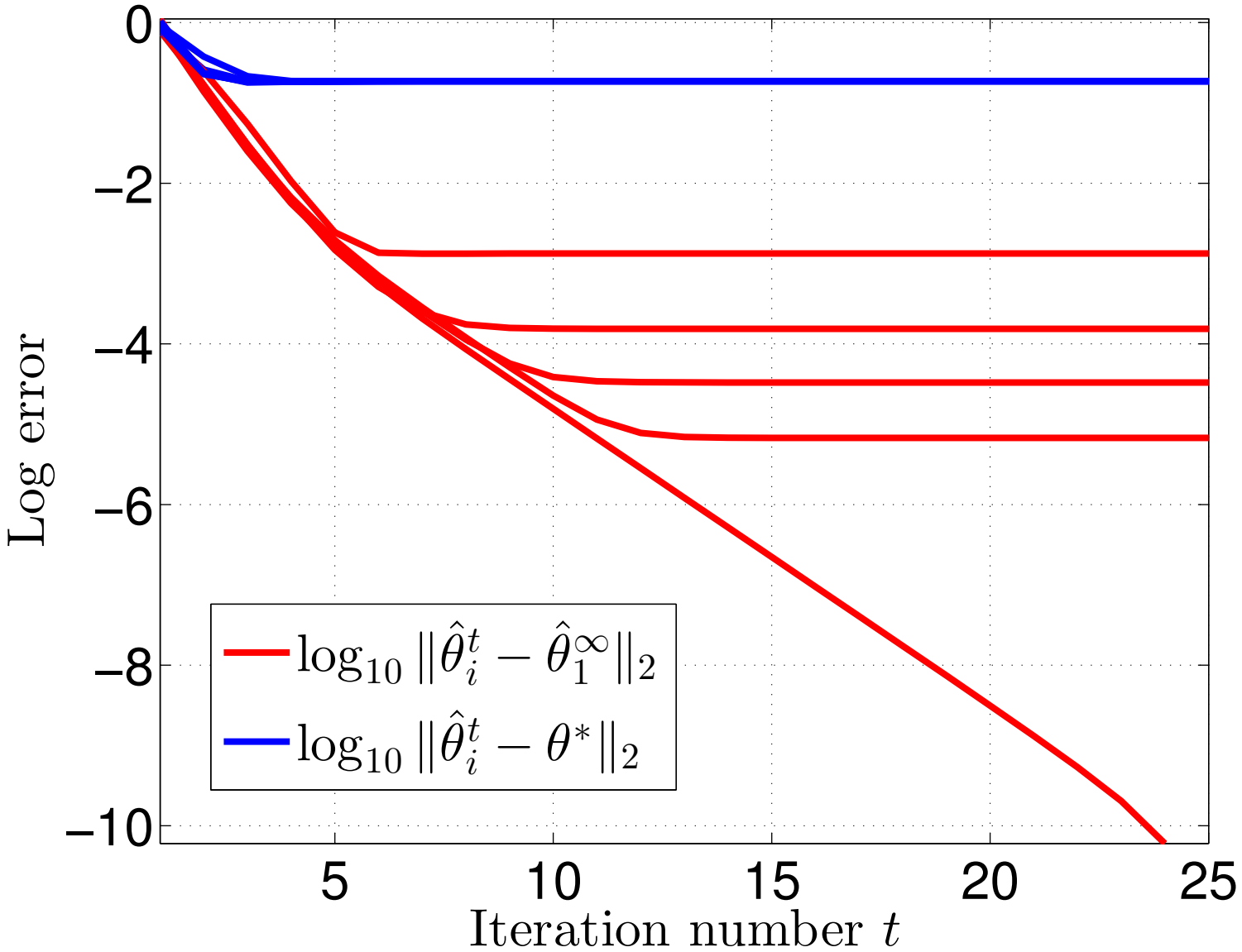
Baum-Welch algorithm - empirical observations



Baum-Welch algorithm - theoretical guarantees

Regularity conditions:

- "Lipschitz condition" on $Q = \mathbb{E}Q_n$, i.e.
  $\sup_\theta \|\nabla_\theta Q(\theta \mid \theta') - \nabla_\theta Q(\theta \mid \theta^*)\|_2 \leq L\|\theta' - \theta^*\|_2$
- Strong concavity of Q, i.e.
  $Q(\theta_1) - Q(\theta_2) \leq \nabla Q(\theta_2)^T(\theta_1 - \theta_2) - \frac{\lambda}{2}\|\theta_1 - \theta_2\|_2^2$

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Proof idea - Additional steps for dependent data

Based on a framework by Balakrishnan et al. 2014 for i.i.d. data

For example $\mathbb{E}Q_n$ depends on n and it is a sum of dependent random variables:

$$Q_n(\theta \mid \theta') \sim \frac{1}{n}\mathbb{E}_{Z_1^n|X_1^n,\theta'}\left[\log p(X_1^n, Z_1^n; \theta)\right]$$
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- Use that far away blocks $\{X_{i-k}^{i+k}\}_{i=1,\ldots,n}$ are almost independent

Guarantees for Gaussian output HMM

Model assumptions

- $Z_i \in \{-1, +1\}$ is a mixing Markov chain with symmetric transition matrix
- Normal observation densities $X_i \mid Z_i \sim \mathcal{N}(Z_i \mu^*, \sigma^2 I)$.

Defining the SNR $\eta^2 = \frac{\|\mu^*\|_2^2}{\sigma^2}$ we have

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Corollary (Y., Balakrishnan and Wainwright '15)

Given $n \gtrsim d \log^2(d/\delta)$ *and the initialization* $\hat{\mu}^0 \in \mathbb{B}_2(\frac{\|\mu^*\|_2}{4}; \mu^*)$, *we obtain*

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$$\|\widehat{\theta}^t - \theta^*\|_2 \leq \kappa^t \|\widehat{\theta}^0 - \theta^*\|_2 + \frac{C}{1-\kappa}\Big[\sigma\sqrt{\frac{d\log^2(\frac{n}{\delta})}{n}} + \|\mu^*\|_2\sqrt{\frac{\log^2(\frac{n}{\delta})}{n}}\Big]$$

with probability at least $1 - \delta$ *and* $\kappa \propto \mathrm{e}^{-c\eta^2} \log d$.

Guarantees for Gaussian output HMM

Model assumptions

- $Z_i \in \{-1, +1\}$ is a mixing Markov chain with symmetric transition matrix
- Normal observation densities $X_i \mid Z_i \sim \mathcal{N}(Z_i \mu^*, \sigma^2 I)$.

Defining the SNR $\eta^2 = \frac{\|\mu^*\|_2^2}{\sigma^2}$ we have

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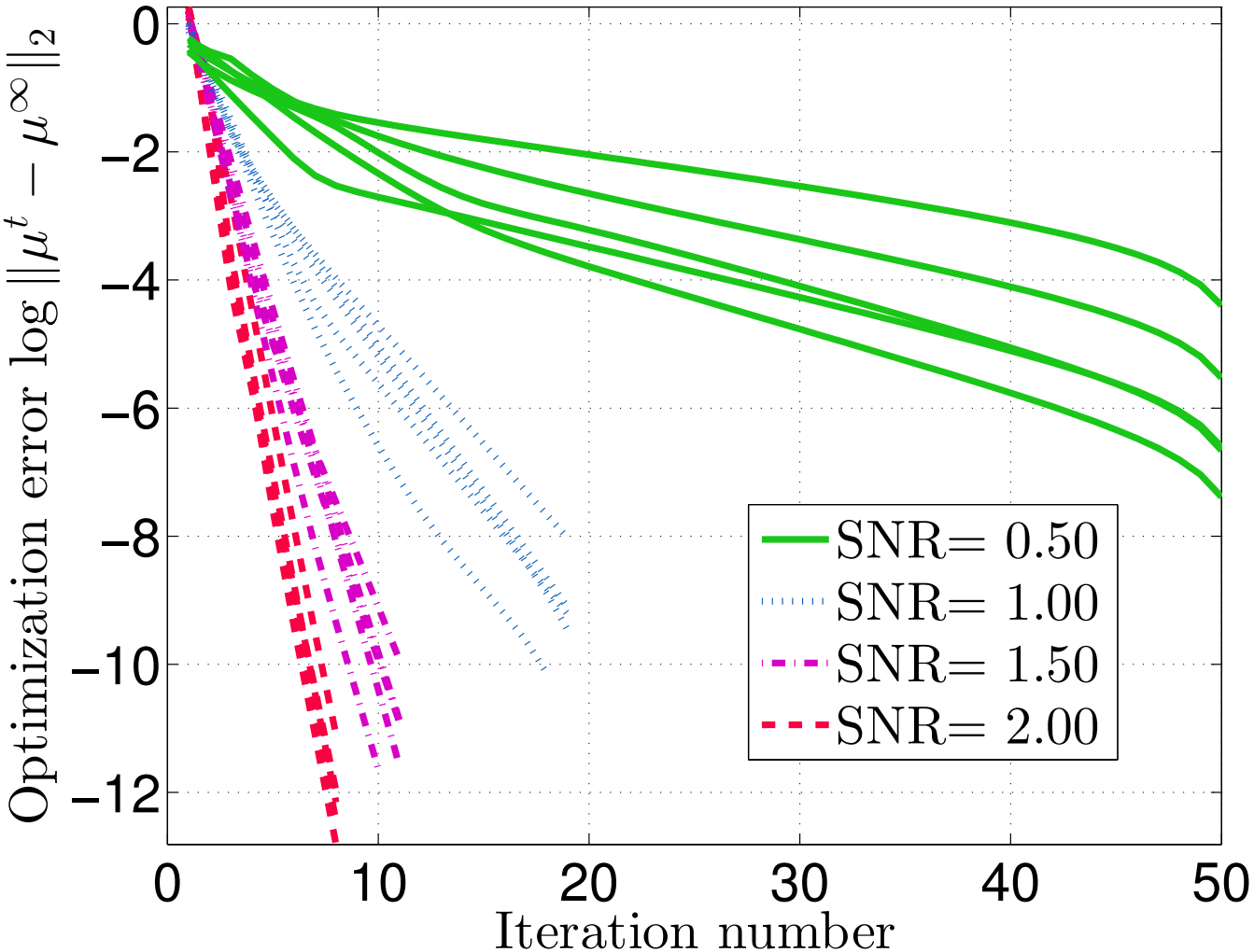
Convergence rate dependence on SNR

Parameter settings: $d = 10, n = 1000, \sigma = 2$



Contributions

In this work we

- guarantee that the Baum-Welch estimate gets close to θ^*
- characterize the basin of attraction depending on the model
- give a linear convergence rate of the BW algorithm in this basin
- provide explicit values for the Gaussian output HMM example

Future work

Some open questions

- How hard is it to compute Lipschitz constants for models where the output distribution is not an exponential family?
- Can we extend the framework to more general graphical models like lattices etc.?

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THANK YOU!